Section A Notes

Signal:- Dependent variable or function of one or more independent variables.

$$f(x_1, x_2 \,--\, x_n)$$

f ↓ Signal

$x_1, x_2 \ldots x_n$ ↳ independent variable.

Example:- A.C → is a signal because current is changing with change in time.

D.C → is not a signal there is a change in current is contant or current is constant

1) **Single variable signal / 1D signal:-**
Is a signal which depend upon single variable or it is a function of only one variable.
Ex:- $f(x)$, $g(x)$ etc

2) **Multivariable signal / 2D signal:-**
These are the signal which depends upon more than one variable
Ex:- $f(x_1, x_2)$, $f(x_1, x_2, x_3)$ etc.

Classification of signal

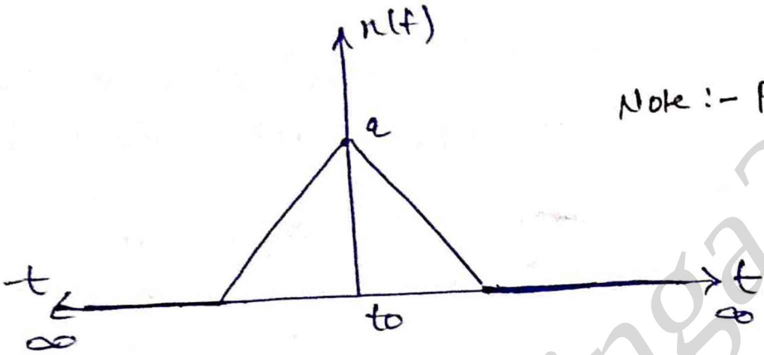
- Continuous time signal
- Discrete time signal

Continuous time signal :- Specified for every value of time
[CTS]

Representation :- $x(t)$

x → dependent variable

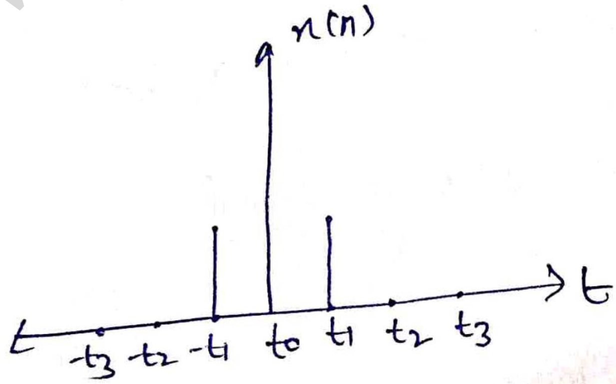
t → Independent variable.



Note :- Practically the signal is inclining with time, because you can't reverse the time.

Discrete Time Signal :- Specified at discrete (certain or) time intervals. It is not specified for every value to time (t)
[DTS]

Representation → $x(n)$ where n = integer, x = function / d. variable.



$\Delta t = t_1 - t_0$.

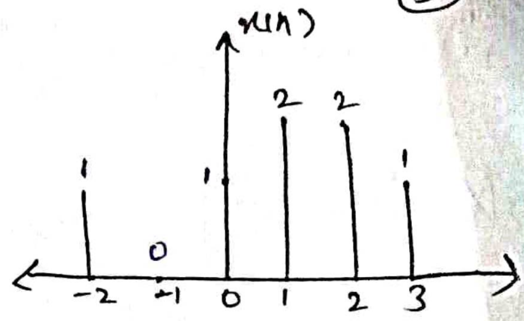
If $\Delta t = t_1 - t_0 = t_2 - t_1 = t_3 - t_2 = t_n - t_{n-1}$

then if say that the discrete time signal is uniformly sampled.

Example A discrete time signal.

$$x(n) = \{\overset{-2}{1}, \overset{-1}{0}, \underset{\uparrow}{\overset{0}{1}}, \overset{+1}{2}, \overset{2}{2}, \overset{3}{1}\}$$

$x(n)$ when $n=0$



Addition of Continuous Time Signal

$x_3(t) = x_1(t) + x_2(t)$

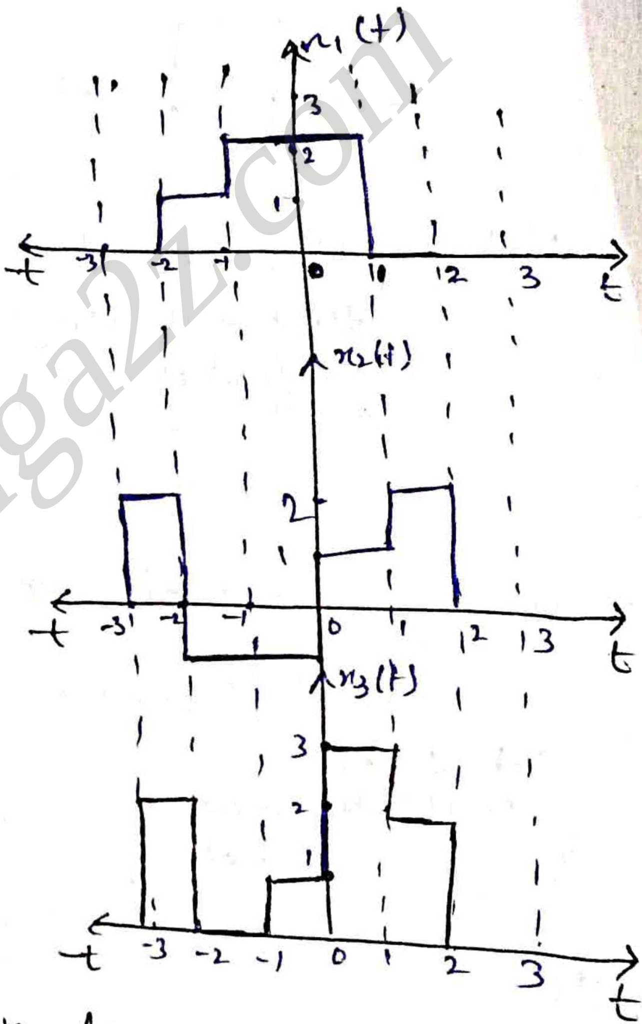
$1 < t \le 2$
$t = 2$
$x_1(t) = 0$
$x_2(t) = 2$
$\therefore x_3(t) = 0+2 = 2$

$0 < t \le 1$
$t = 1$
$x_1(t) = 2$
$x_2(t) = 1$
$x_3(t) = 2+1 = 3$

$-3 < t \le -2$
$x_1(t) = 0$
$x_2(t) = 2$
$x_3(t) = 2+0 = 2$

$-1 < t \le 0$
$x_1(t) = 2$
$x_2(t) = -1$
$\therefore x_3(t) = 2+(-1) = 1$

$-2 < t \le -1$
$x_1(t) = +1$
$x_2(t) = -1$
$\therefore x_3(t) = 0$



Time scaling of continuous Time signal

Scaling
- Time Scaling
- Amplitude scaling.

Time Scaling :— The compression or expansion of a signal in time

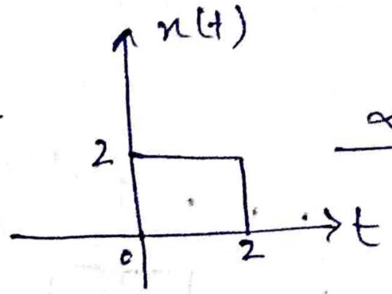
In time scaling we multiply the time with number which is not equal to zero.

$\therefore \; x(t) \xrightarrow[\alpha]{T.S} y(t) = x(\alpha t) \quad \alpha \neq 0$

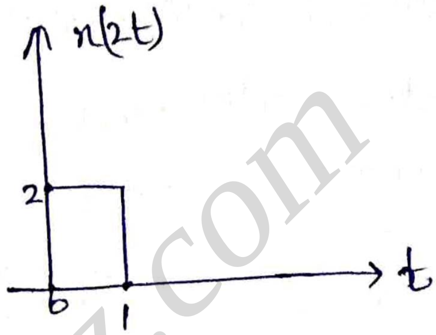
$\alpha \downarrow$ no. of multiplier.

☆ $\boxed{\text{Case - I}}$ $\quad |\alpha| > 1 \quad \alpha \in (-\infty, -1) \cup (1, \infty)$

$x(t) = 2$
$0 \le t \le 2$



$\xrightarrow{\alpha = 2} x(2t)$



Note:- Here we have seen that the compression of the signal of waveform

When $t = 0$

$x(2.0) = x(0) = 2$

When $t = 1$

$x(2.1) = x(2) = 2$

$x(2t) = 2$

case-2 $\quad |\alpha| < 1 \quad \alpha \in (-1, 0) \cup (0, -1)$

$\therefore \; \alpha = 0.1, 0.2 \text{ ---- etc}$

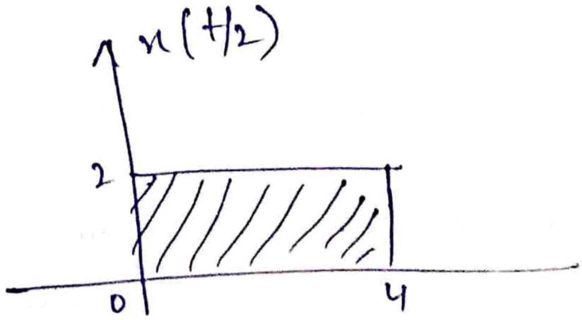
let $\alpha = 0.5$

i) Amplitude same

ii) $\dfrac{\alpha t}{\alpha} = \dfrac{0.5 \times t}{0.5} = t$

(iii) $\dfrac{2}{0.5} = 4$

(iv) $\dfrac{0}{0.5} = 0.$



Here we have seen the case expansion of the same above Question.

Amplitude Scaling

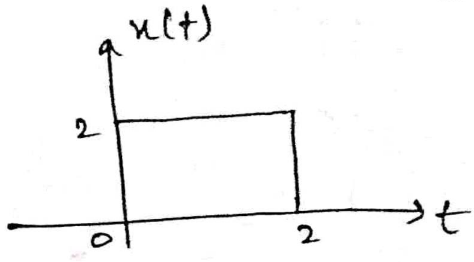
→ In case of amplitude scaling we multiply the amplitude of signal by a real number.

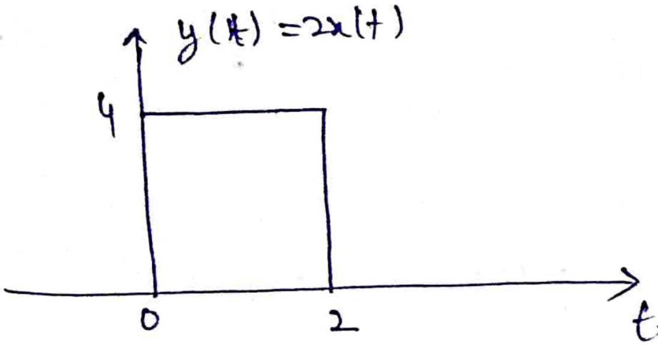
$x(t) \xrightarrow[\beta]{A.S} y(t) = \beta x(t)$

$-x(t)$

Case-I $|\beta| > 1$ $\quad \beta \in (-\infty,-1) \cup (1,\infty)$

$$x(t) = \begin{cases} 0 & t<0 \\ 2 & 0 \le t \le 2 \\ 0 & t>0 \end{cases}$$



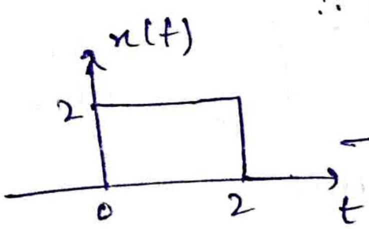


$\xleftarrow{}$ A·S
$\beta = 2$
$|\beta| = 2 > 1$

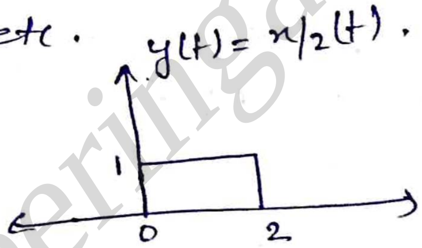
→ This show the amplification in the wave form.

Case-II $|\beta| < 1$ $\quad \beta \in (-1,0) \cup (0,1)$

$\therefore \beta = 0.1,\ 0.7$ etc.



$\xrightarrow[\beta = 0.5]{A \cdot S}$



This shows the case of reduction.

Shifting of continuous Time Signal

Shifting < Time shifting / Amplitude shifting.

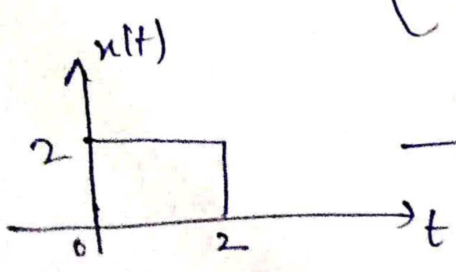
Time shifting

Let $x(t) \xrightarrow[K]{T.S} y(t) = x(t+K)$ sec.

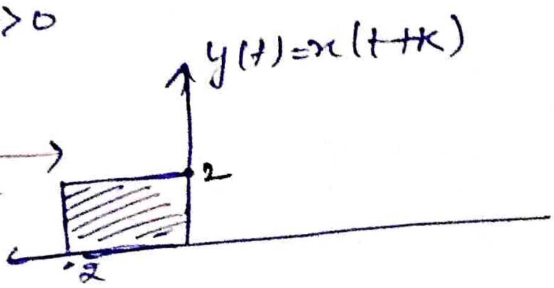
$x(0) = 2 = y(-2)$ $x(2) = 2 \quad y(0)$ The whole wave is shifted left and it is case of time advance

Case-I $K > 0$ $\quad (K \longrightarrow +ve)$

$$x(t) \begin{cases} 0 & t<0 \\ 2 & 0 \le t \le 2 \\ 0 & t>0 \end{cases}$$

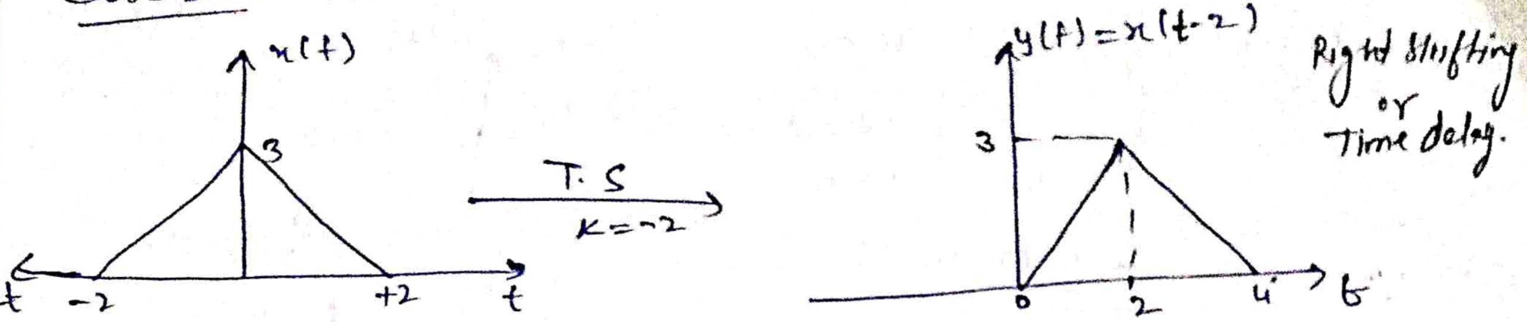


$\xrightarrow[K=+2]{T.S}$



$\cdot -x(t)$

Case - II $K < 0$ ($K \rightarrow -ve$)



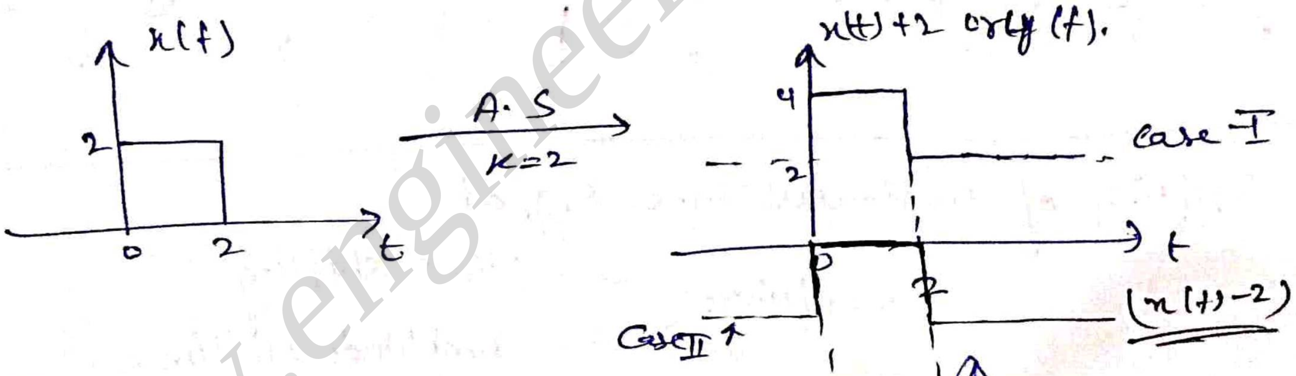
Amplitude shifting

$$x(t) \xrightarrow[K]{A \cdot S} y(t) = x(t) + K.$$

Case - I $K > 0$

$K = +2$

$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases} \quad y(t) = x(t) + 2 = \begin{cases} 2 & t < 0 \\ 4 & 0 \le t \le 2 \\ 2 & t > 2 \end{cases}$$



Case - II $K < 0$

$K = -2$

$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases} \quad y(t) = x(t) - 2 \quad \begin{cases} -2 & t < 0 \\ 0 & 0 \le t \le 2 \\ -2 & t > 2 \end{cases}$$

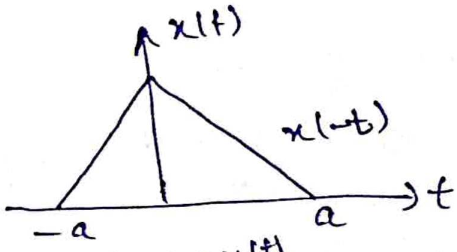
Even Signal :- Remain identical under folding operation or reflection / Time-Reversal.

The signal which are symmetrical or mirror image about the Y axis are called as even signals

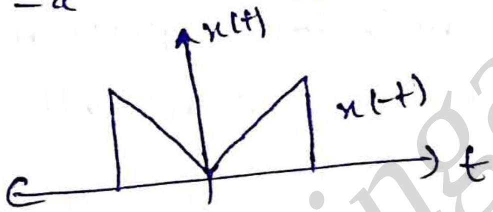
$$x(t) \xrightarrow{T.R} x(-t) = x(t)$$

Examples

i)



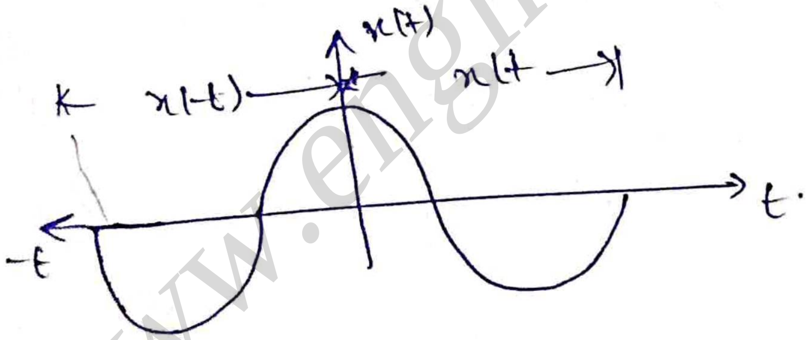
ii)



iii) $x(t) = \cancel{\cos\omega t}\ \cos\omega t$

$t \longrightarrow -t$

$x(-t) = \cos(-\omega t) = \cos(\omega t) = x(t)$

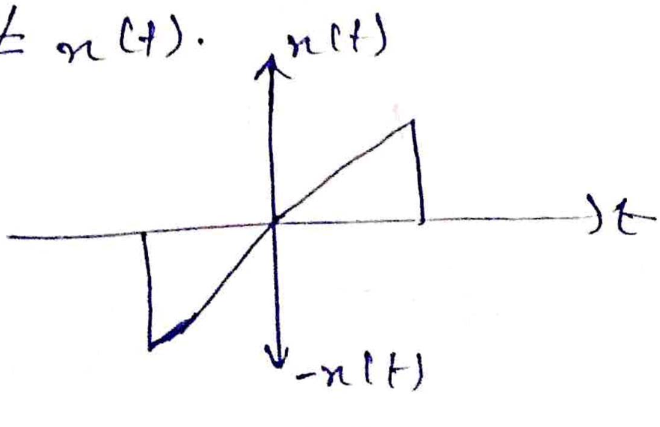


→ **ODD Signal** :- Doesn't remain identical under folding operation.

$$x(-t) \neq x(t).$$

$x(-t) = -x(t)$

or $x(t) = -x(-t)$.



when $t = 0$

$x(0) = -x(-0)$

$x(0) = -x(0)$.

$x(0) = 0$.

Properties:

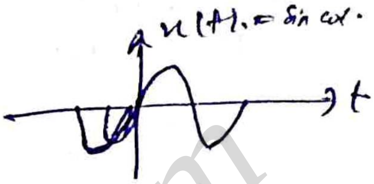
i) $t=0$ odd signal $= 0$

ii) average / mean / d.c value of odd signal $= 0$

(iii) $x(t) = \sin \omega t$ $\sin(-\theta) = -\sin\theta$

$x(-t) = -\sin \omega t$

$x(-t) = -x(t)$.



Even & odd components of signal.

>> Any continuous time signal can be represented as sum of odd & even components.

>> A general signal is neither even nor odd but have both even & odd components.

$x(t) \longrightarrow$ C.T. signal

$x_e(t) \longrightarrow$ even comp. of signal $x(t)$

$x_o(t) \longrightarrow$ odd " " "

$x(t) \longrightarrow x_e(t) + x_o(t) \longrightarrow (1)$

Put $t = -t$

$x(-t) \longrightarrow x_e(-t) + x_o(-t)$.

$x(-t) \longrightarrow x_e(t) - x_o(t) \longrightarrow (2)$

(1) + (2)

$x(t) + x(-t) = 2x_e(t)$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

(1) - (2)

$x(t) - x(-t) = 2x_o(t)$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

PERIODIC AND NON-PERIODIC

Periodic Signal :- A signal which repeats itself after a fixed time period.

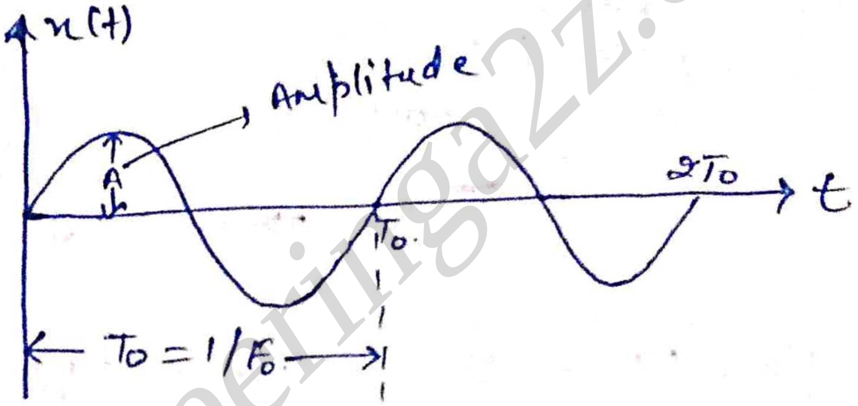
Mathematically :- $x(t) = x(t+T_0)$

where T_0 — period of signal.

OR

$x(t)$ repeats itself after a period of T_0 sec

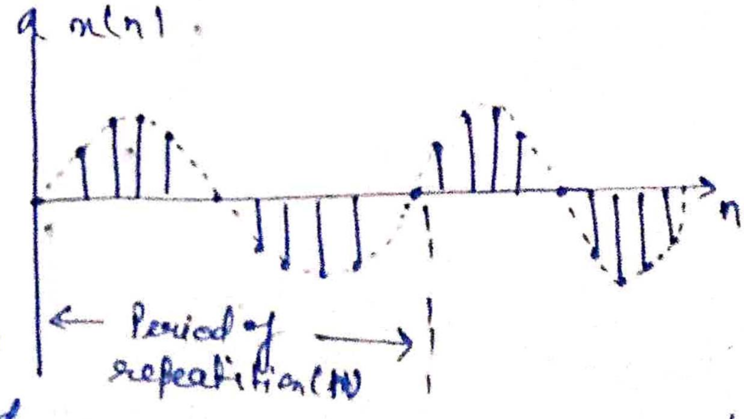
Example :- Sine, cosine waves, square wave etc.



(b) **Periodic Discrete-Time Signal**

$x(n) = x(n+N)$

Note :- The smallest value of N for which the condition of periodicity exists is called fundamental period



Here N is the period of signal.

<u>Non-Periodic Signal</u> :- A signal which does not repeat itself after a fixed period and do not repeat at all called a non-periodic signal

$$x(t) \neq x(t+T_0).$$

In Normal it has $T_0 = \infty$ (decaying exponential signal)

$x(t) = e^{-\alpha t}$

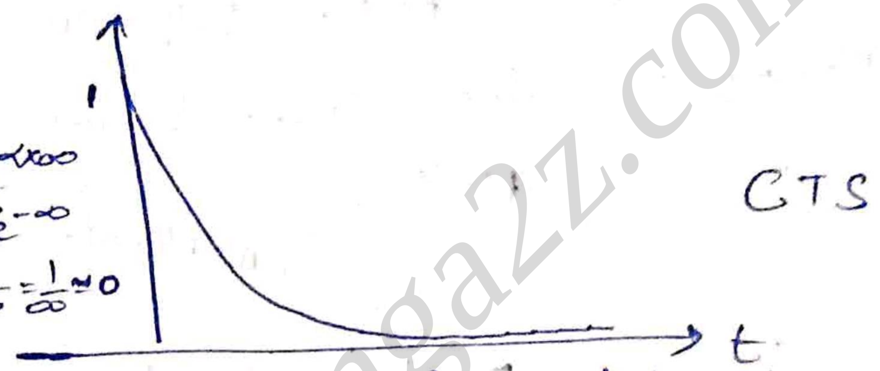
If $t=0$, $t=\infty$

$x(t) = e^{-\alpha \times 0}$ $x(t) = e^{-\alpha \times \infty}$

$x(t) = e^{0} = 1$ $x(t) = e^{-\infty}$

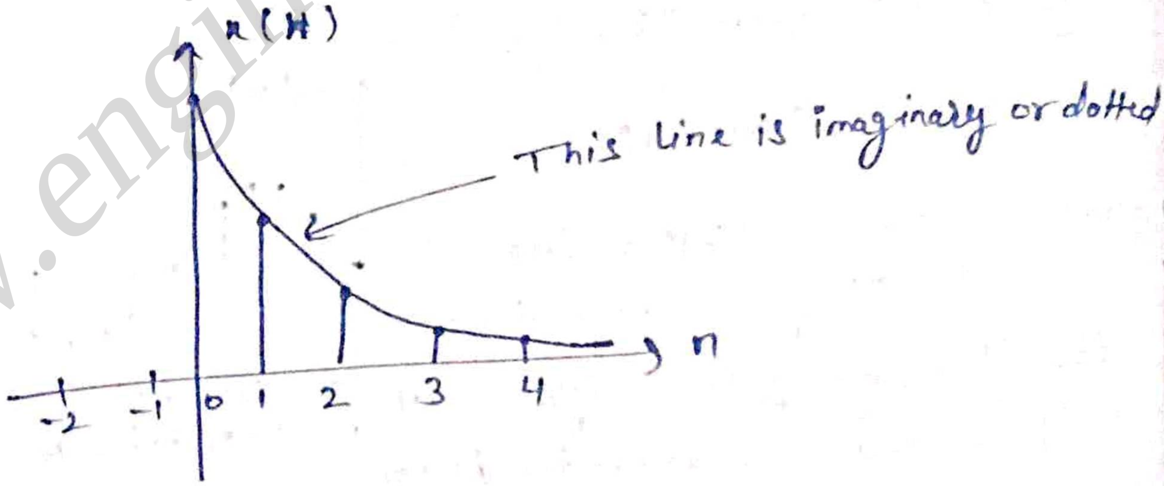
$x(t) = \frac{1}{e^{\infty}} = \frac{1}{\infty} \approx 0$

$\boxed{\frac{1}{e^{\infty}} \sim 0}$



$T_0 = \infty$ (one cycle period).

<u>Example</u> :- DC signal, rectangular signal etc.



<u>Note</u> :- A discrete time sinusoidal signal is periodic only if its frequency f_0 is rational. It means that f_0 should be in P/q form where P & q are integer.

ENERGY & POWER SIGNALS. (11)

A signal is classified as a power or energy signal based on its average power or total energy.

→ The <u>energy signal</u> is one which has finite energy and zero average power

Non-periodic signal are energy signal

$x(t)$ = energy signal if $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$.

Imp $\boxed{0 < E < \infty \text{ and } P_{av} = 0.}$

→ The power signal is one which has finite average power and infinite energy.

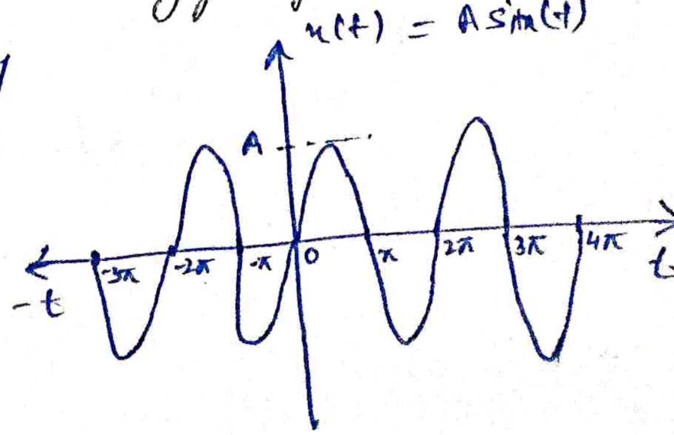
$x(t)$ is power signal if

Imp. $\boxed{0 < P < \infty \ \& \ \text{Energy}(E) = \infty}$

Expression $= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

Note:- Practically periodic signals are power signal but converse is not true.

Example :- Sketch the following signal $x(t) = A \sin t$ for $-\infty < t < \infty$ And check whether the signal is power or an energy signal.



Sol:- By seeing the fig. we can say that signal is periodic. Hence it is power signal.

$T = 2\pi$ (FTP)

We know that

$$P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

Here $x(t) = A\sin t$

So. $T = 2\pi$

$$P = \frac{1}{2\pi}\int_{-\pi}^{\pi} |A\sin t|^2 dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} A^2 \sin^2 t \, dt$$

$$P = \frac{A^2}{2\pi}\int_{-\pi}^{\pi} \sin^2 t \, dt = \frac{A^2}{2\pi}\int_{-\pi}^{\pi} \frac{1-\cos 2t}{2} dt$$ (using trigo.)

$$P = \frac{A^2}{2\pi}\left[\int_{-\pi}^{\pi} \frac{1}{2} dt - \int_{-\pi}^{\pi} \frac{\cos 2t}{2} dt\right]$$

$$P = \frac{A^2}{2\pi}\left[\left[\frac{t}{2}\right]_{-\pi}^{\pi} - \left(\frac{\sin 2t}{2} \times \frac{1}{2}\right)_{-\pi}^{\pi}\right]$$

$$P = \frac{A^2}{2\pi}\left[\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) - \left(\frac{\sin 2\pi}{4} - \left(-\frac{\sin 2\pi}{4}\right)\right)\right]$$

$$P = \frac{A^2}{2\pi}\left\{[\pi] - \left(\frac{0}{4} + \frac{0}{4}\right)\right]$$

$$P = \frac{A^2}{2\pi}\left\{\pi - 0\right\}$$

$$P = \frac{A^2}{2\pi} \times \pi \Rightarrow \frac{A^2}{2} \quad \text{Ans}$$

Note :— $\boxed{\text{Power signals can exist over infinite time. But Energy signal are time limited.}}$ Imp. point.

$\therefore$ P ranges time $= -\infty < t < \infty$

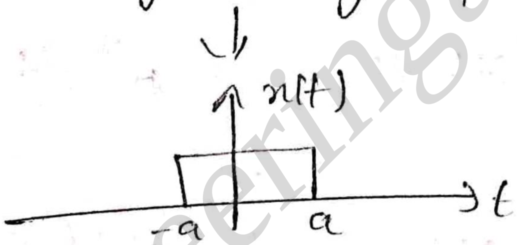
E ranges time $= -\alpha < t < \alpha$ where α is integer

Point to Remember for Energy Signal

- Non-periodic Signals are energy signal
- These signal are time limited eg $-\alpha < t < \alpha$ where α is integer
- Power of Energy signal is zero $(P = 0)$
- Energy signal have finite value of Energy. $0 < E < \infty$
- $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$ for Continuous time signal (CTS)

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$ for discrete time signal (DTS)

- Example :- a Single rectangular pulse



$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

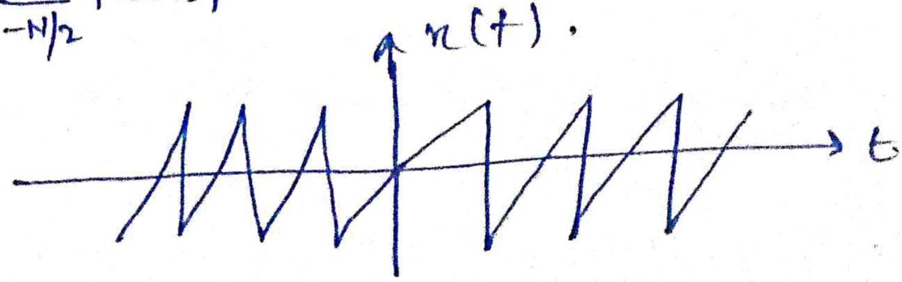
$$e^{j\theta} = \cos\theta + j\sin\theta$$

⇒ Points to Remember for Power Signal

- Practical periodic signal are power signals.
- These signal can exist over infinite time
- Energy of Power signal is infinite. $\boxed{E = \infty}$
- Power signal have power finite value. $0 < P < \infty$
- $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$ OR $\frac{1}{2T} \int_{-T}^{+T} x^2(t)\, dt$. (CTS)

$$P = \lim_{N \to \infty} \frac{1}{N} \sum_{-N/2}^{N/2} |x(n)|^2$$

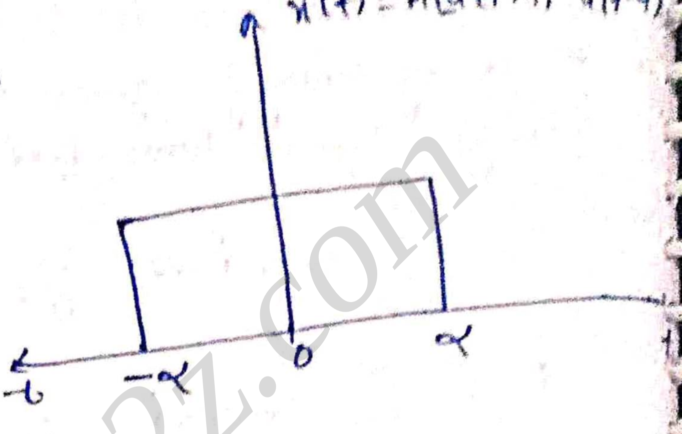
Example



Example 2 Sketch the following signal:-

$x(t) = A\,[u(t+\alpha) - u(t-\alpha)]$ for $\alpha > 0$.

Also determine whether the given signal is power signal or an energy signal or neither.



Here it is clear that $x(t)$ has a finite duration signal

$\therefore$ $x(t)$ is energy ~~F~~ signal.

$\Rightarrow$ We know that

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_{-\alpha}^{\alpha} x^2(t)\, dt = \int_{-\alpha}^{\alpha} A^2\, dt$$

As $x(t) = A$ for $-\alpha < t < \alpha$

$$E = \int_{-\alpha}^{0} A^2 dt + \int_{0}^{\alpha} A^2 dt$$

$$\Rightarrow 2\int_{0}^{\alpha} A^2 dt \Rightarrow 2A^2 \int_{0}^{\alpha} 1\, dt$$

$$\Rightarrow 2A^2\,[t]_0^{\alpha} \Rightarrow 2A^2(\alpha - 0)$$

$$\Rightarrow \boxed{2\alpha A^2} \text{ Ans.}$$

(i) Prove that

(a) The power the Energy signal is zero over infinite time.

$$P = \lim_{T\to\infty} \frac{T}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$\Rightarrow \underset{T\to\infty}{Lt} \frac{1}{T}\left[\underset{T\to\infty}{Lt} \int_{-T/2}^{T/2} x^2(t)\, dt = \underset{T\to\infty}{Lt} \frac{1}{T}\left[\int_{-\infty}^{\infty} x^2(t)\, dt\right]\right.$$

$$P \Rightarrow \underset{T\to\infty}{Lt} \frac{1}{T} \times E \qquad \left(\because E = \int_{-\infty}^{\infty} x^2(t)\, dt\right)$$

$$P = \frac{E}{\infty} = 0 \times E = 0 \qquad \left(\underset{x\to\infty}{Lt} \frac{1}{x} = 0\right)$$

Thus Power of Energy Signal is zero over infinite time.

(b) The Energy of power Signal is infinite over infinite time. Prove that?

We know that

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \underset{T\to\infty}{Lt} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$E = \underset{T\to\infty}{Lt}\ T \cdot \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \qquad \left(\text{Multiply \& divide By } T\right)$$

$$E = T \lim_{T\to\infty} \left[\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \right]$$

$$E = \lim_{T\to\infty} . T \left[P \right]$$

$$E = \lim_{T\to\infty} T \times P \quad \left[\text{Put } T = \infty \right.$$

$$E = \infty \times P$$

$$E = \infty$$

Thus Energy of power signal is infinite over infinite time.